Lecturered By Dr. H.K. Yadav.
Deptt. of Mathematics
Marwari College, Darbhanga.
I.Sc. Class-XI Date-23-04-2020

Miscellaneous (permutations and combinations)

Q.1. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and two different consonants can be formed from the alphabet?

Soln. Since, 2 vowels out of 5 vowels can be selected in $^5C_2$ ways. and 2 consonants out of 21 consonants can be selected in $^{21}C_2$ ways. Also, these 4 alphabets (2 vowels and 2 consonants) can be arranged in $\lfloor 4$ ways.

Hence, by Fundamental principle of Counting, total number of ways $= {}^5C_2 \times {}^{21}C_2 \times \lfloor 4 = \frac{\lfloor 5}{\lfloor 2 \lfloor 5-2} + \frac{\lfloor 21}{\lfloor 2 \lfloor 21-2} \times \lfloor 4$

$= \frac{\lfloor 5}{\lfloor 2 \lfloor 3} + \frac{\lfloor 21}{\lfloor 2 \lfloor 19} \times \lfloor 4 = \frac{5 \times \cancel{4}^2 \times \cancel{\lfloor 3}}{\cancel{2} \times 1 \times \cancel{\lfloor 3}} \times \frac{21 \times 20 \times \cancel{\lfloor 19}}{2 \times 1 \times \cancel{\lfloor 19}} \times 4 \times 3 \times 2 \times 1$

$= 10 \times 420 \times 12 = 120 \times 420 = 50400$

Q.2. In an examination, a question paper consists of 12 questions divided into two parts i.e. part I and part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all selecting atleast 3 from each part. In how many ways can a student select the questions.

Soln: We have to attempt 8 questions keeping in mind that atleast 3 from each part, then following possibility arises.
Now, 3 questions out of 5 questions and 5 questions out of 7 questions can be selected ~~in~~ part I and part II respectively
$= {}^5C_3 \times {}^7C_5$ —— (i)
4 questions out of 5 questions and 4 questions out of 7 questions can be selected part I and part II respectively
$= {}^5C_4 \times {}^7C_4$ —— (ii)
Also, 5 questions out of 5 questions and 3 questions out of 7 questions can be selected part I and part II respectively.
$= {}^5C_5 \times {}^7C_3$
Hence, by fundamental principle of counting total number of ways $= ({}^5C_3 \times {}^7C_5) + ({}^5C_4 \times {}^7C_4) + ({}^5C_5 \times {}^7C_3)$

g. Sc. Class - XI , Date - 23-04-2020.

Rest part of Q. 2.

$$= \left(\frac{5!}{3!\,(5-3)!} \times \frac{7!}{5!\,(7-5)!}\right) + \left(\frac{5!}{4!\,(5-4)!} \times \frac{7!}{4!\,(7-4)!}\right) + \left(\frac{5!}{5!\,(5-5)!} \times \frac{7!}{3!\,(7-3)!}\right)$$

$$= \left(\frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!}\right) + \left(\frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!}\right) + \left(\frac{5!}{5! \times 0!} \times \frac{7!}{3!\,4!}\right)$$

$$= \left(\frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{7 \times 6 \times 5!}{5! \times 2 \times 1}\right) + \left(\frac{5 \times 4!}{4! \times 1} \times \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1}\right) + \left(\frac{5!}{5! \times 1} \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!}\right)$$

$$= \left(\frac{5 \times 4}{2} \times \frac{7 \times 6}{2}\right) + \left(\frac{5}{1} \times \frac{7 \times 6 \times 5}{3 \times 2}\right) + \left(1 \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1}\right)$$

$= 210 + 175 + 35 = 210 + 210 = 420$ ways.

Q. 3 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Soln. Since, total number of persons = 9

1, 2, 3, 4, 5, 6, 7, 8, 9

Here, we see that 4 even numbers and 4 women want to sit, this can be done in $4!$ ways.
Remaining 5 seats can be filled by 5 men in $5!$ ways.

Hence, by Fundamental Principle of Counting total number of ways

$= 5! \times 4!$

$= 5 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1$

$= 120 \times 24$

$= 2880$ ways.

$$\begin{array}{r} 24 \\ 12 \\ \hline 48 \\ 24 \\ \hline 288 \end{array}$$

Scanned with CamScanner