

Lecturered By Dr. H.K. Yadav

Deptt. of mathematics

Mamari College, Darbhanga.

J.Sc. Class-XI Date-23-04-2020

Miscellaneous (permutations and combinations)

Q.1. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and two different consonants can be formed from the alphabet?

Soln: Since, 2 vowels out of 5 vowels can be selected in 5C_2 ways. and 2 consonants out of 21 consonants can be selected in ${}^{21}C_2$ ways. Also, these 4 alphabets (2 vowels and 2 consonants) can be arranged in 4! ways.

Hence, by Fundamental principle of Counting, total number of ways

$$\begin{aligned} &= {}^5C_2 \times {}^{21}C_2 \times 4! = \frac{5!}{2!3!} \times \frac{21!}{2!19!} \times 4! \\ &= \frac{5!}{2!3!} \times \frac{21!}{2!19!} \times 4! = \frac{5 \times 4 \times 3}{2 \times 1} \times \frac{21 \times 20}{2 \times 1} \times 4 \times 3 \times 2 \times 1 \\ &= 10 \times 420 \times 12 = 120 \times 420 = 50400 \end{aligned}$$

Q.2. In an examination, a question paper consists of 12 questions divided into two parts i.e. part I and part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all selecting atleast 3 from each part. In how many ways can a student select the questions.

Soln: We have to attempt 8 questions keeping in mind that atleast 3 from each part, then following possibility arises. Now, 3 questions out of 5 questions and 5 questions out of 7 questions

can be selected in part I and part II respectively

$$= {}^5C_3 \times {}^7C_5 \quad \text{--- (i)}$$

4 questions out of 5 questions and 4 questions out of 7 questions can be selected part I and part II respectively

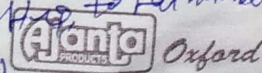
$$= {}^5C_4 \times {}^7C_4 \quad \text{--- (ii)}$$

Also, 5 questions out of 5 questions and 3 questions out of 7 questions can be selected part I and part II respectively.

$$= {}^5C_5 \times {}^7C_3$$

Hence, by fundamental principle of counting, total number of ways

$$= ({}^5C_3 \times {}^7C_5) + ({}^5C_4 \times {}^7C_4) + ({}^5C_5 \times {}^7C_3)$$



Rest part of Q.2.

$$\begin{aligned}
 &= \left(\frac{15}{13 \times 12} \times \frac{17}{11 \times 10} \right) + \left(\frac{15}{14 \times 13} \times \frac{17}{12 \times 11} \right) + \left(\frac{15}{15 \times 14} \times \frac{17}{13 \times 12} \right) \\
 &= \left(\frac{15}{13 \times 12} + \frac{17}{11 \times 10} \right) + \left(\frac{15}{14 \times 13} + \frac{17}{12 \times 11} \right) + \left(\frac{15}{15 \times 14} + \frac{17}{13 \times 12} \right) \\
 &= \left(\frac{5 \times 4 \times 15}{13 \times 12 \times 11} + \frac{4 \times 6 \times 15}{13 \times 12 \times 11} \right) + \left(\frac{15 \times 17}{14 \times 13 \times 11} + \frac{7 \times 6 \times 5 \times 17}{14 \times 13 \times 11 \times 10} \right) \\
 &\quad + \left(\frac{15}{15 \times 14} + \frac{7 \times 6 \times 5 \times 17}{15 \times 14 \times 13 \times 12} \right) \\
 &= \left(\frac{5 \times 4 \times 15}{13 \times 12 \times 11} + \frac{7 \times 6}{11 \times 10} \right) + \left(\frac{5}{1} + \frac{7 \times 6 \times 5}{13 \times 12} \right) + \left(\frac{1 \times 7 \times 6 \times 5}{13 \times 12 \times 11} \right) \\
 &= 210 + 175 + 35 = 210 + 210 = 420 \text{ ways.}
 \end{aligned}$$

Q. ③ It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Soln. Since, total number of persons = 9

1, 2, 3, 4, 5, 6, 7, 8, 9

Here, we see that 4 even numbers and 4 women want to sit, this can be done in 14 ways.

Remaining 5 seats can be filled by 5 men in 15 ways.

Hence, by Fundamental principle of Counting, total number of ways

$$\begin{aligned}
 &= 15 \times 14 \\
 &= 5 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 \\
 &= 120 \times 24 \\
 &= 2880 \text{ ways.}
 \end{aligned}$$

$$\begin{array}{r}
 24 \\
 12 \\
 \hline
 248 \\
 24 \\
 \hline
 288
 \end{array}$$